Correlation functions of the ABJM model

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ABSTRACT

In the ABJM model, we study the two point function of various solitonic strings and the three point functions with a marginal or (ir)relevant scalar operator by using the AdS/CFT correspondence. After explicitly calculating these quantities, we show that the string results are perfectly consistent with the RG analysis of the dual gauge theory. Especially, for the circular string wrapped in ϕ_1 we find a new size effect like the finite size effect of the magnon, which may describe the size effect of the closed spin chain. Finally, we calculate the three point function with an (ir)relevant scalar operator, which is closely related to that with a marginal operator.

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1 Introduction

Applying of the AdS/CFT correspondence to strongly interacting system is one of the active research areas of the theoretical physics. In order to understand in depth such duality and the gauge theory in the strong coupling regime, we need to know more clearly the underlying structure of the AdS/CFT correspondence. One of good examples to understand the AdS/CFT correspondence is the 4-dimensional $\mathcal{N} = 4$ SYM theory dual to the string theory or supergravity on $AdS_5 \times S^5$, in which the conformal symmetry and the integrability play a crucial role to know the physics of the strong interacting theory [1]-[13]. Recently, such works has been generalized to other dimensions. For example, in order to investigate the world volume theory of M-brane, the 3-dimensional $\mathcal{N} = 8$ Bagger-Lambert-Gustavsson (BLG) model and the Aharony-Bergman-Jafferis-Maldacena (ABJM) model describing the $\mathcal{N} = 6$ Chern-Simons gauge theory have been widely investigated [14]-[33]. Moreover, it was shown that the ABJM model has its dual gravity description defined on $AdS_4 \times CP^3$ and is integrable at least up to the one-loop level [34, 35, 36]. In this paper, we will further investigate the AdS/CFT correspondence of the ABJM model by comparing the various correlation functions of heavy operators, two point and three point functions, evaluated in both string and dual gauge theories [37]-[54]. In addition, we generalize the string theory calculation to the three point function with an (ir)relevant scalar operator, which shows that the AdS/CFT correspondence is very useful and powerful to understand the strongly interacting gauge theory because it is impossible to evaluate the three point function with an (ir)relevant scalar operator in the strong coupling regime.

Related to the conformal symmetry, if we know only two and three point correlation functions, the other higher functions can be determined by them. In general, the coordinate dependence of the correlation functions in the conformal theory is fixed by the global conformal symmetry. So two and three point functions have the following forms

$$\begin{aligned} \langle \mathcal{O}_A(x)\mathcal{O}_B(y)\rangle &= \frac{\delta_{AB}}{|x-y|^{2\Delta}}, \\ \langle \mathcal{O}_A(x)\mathcal{O}_B(y)\mathcal{O}_C(z)\rangle &= \frac{a_{ABC}}{|x-y|^{\Delta_A+\Delta_B-\Delta_C}|x-z|^{\Delta_A+\Delta_C-\Delta_B}|y-z|^{\Delta_B+\Delta_C-\Delta_A}}, \end{aligned} \tag{1}$$

where Δ and a_{ABC} are the conformal dimension and structure constant respectively. If we know the conformal dimension of a primary operator, the two point function can be easily fixed. Although the form of the three point functions is determined by the global conformal symmetry, finding the structure constant is not easy work. Moreover, calculating the structure constant at the strong coupling regime is almost impossible except several cases, in which other symmetries can determined this structure constant even in the strong coupling regime. In this paper, we will consider various heavy operators, which have a very large conformal dimension, and calculate their conformal dimensions by evaluating the semiclassical string partition function. After that, we investigate the three point function between two heavy operators and one marginal scalar operator in both the string theory and dual field theory. In the gauge theory side, if a marginal scalar operator is given by the Lagrangian density operator, the structure constant can be exactly determined by the renormalization group (RG) analysis [37] because the deformed theory by a marginal operator becomes the same theory with a modified gauge coupling. We find that the three point function calculated in the string theory is exactly matched with the RG analysis of the dual field theory. In addition to check the AdS/CFT correspondence between the ABJM model and the string theory on $AdS_4 \times CP^3$, we also suggest a new circular solitoinc string, which would be dual to a closed spin chain of the dual field theory and can show the size effect caused by the change of the closed spin chain length like the finite size effect of the magnon corresponding to open spin chain. For the length of spin chain is very large, the size effect of the closed spin chain is linearly proportional to the size change while the size effect of the open spin chain is exponentially suppressed [12, 20, 24, 53, 54].

As mentioned above, it is very interesting to calculate the general structure constant in the strong coupling regime but difficult in the gauge theory side. Following the AdS/CFT correspondence we can easily evaluate such structure constant even in the strong coupling regime. In this paper, we study the three point function between above heavy operators and an (ir)relevant scalar operator dual to the massive dilaton field propagating on the AdS_4 space. We show that the string theory calculation provides the exactly same coordinate dependence expected by the global conformal symmetry and the structure constant $a_{AA\mathcal{D}_m}$ generated by an (ir)relevant operator is closed related to that with the

marginal one $a_{AA\mathcal{D}_0}$

$$a_{AA\mathcal{D}_m} = \frac{1}{2^{h-2}} \frac{\Gamma\left(\frac{h}{2}\right)\Gamma\left(h\right)}{\Gamma\left(\frac{h+1}{2}\right)\Gamma\left(h-\frac{3}{2}\right)} a_{AA\mathcal{D}_0}, \tag{2}$$

The rest parts are followings: In Sec. 2, we briefly review the RG analysis in the general d-dimensional conformal field theory with a marginal scalar operator [37], in which it will be shown that the structure constant between two general operators and one marginal operator is related to the derivative of the conformal dimension of the general operator with a coefficient depending on the dimensionality. In Sec. 3, we concentrate on various solitonic string configurations of the ABJM model and then investigate the conformal dimensions and three point function with a marginal scalar operator. The results of these calculations show that the string theory calculation are perfectly matched with the previous RG analysis in the dual field theory. Furthermore, we generalize the string calculation of the three point function to ones including an (ir)relevant operator instead of a marginal one in Sec. 4. In the large 't Hooft coupling limit, it is impossible to calculate such three point function in the dual field theory. Our work shows that the three point function with an (ir)relevant operator is closely related to that including a marginal one. Finally, we finish our work with some concluding remarks.

2 RG analysis in the conformal field theory

Recently, the various three- and four-point correlation functions between two heavy and light operators were calculated by using the AdS/CFT correspondence [37, 38, 39, 40]. Especially in the four-dimensional gauge theory, it was found that the three-point function in a CFT is related to the anomalous dimension of the deformed CFT with a marginal operator $\mathcal{D}$ at the CFT fixed point. From the RG analysis, the structure constant of the three point correlation function $a_{\mathcal{L}AA}$ is given by

$$-g^2 \frac{\partial}{\partial g^2} \Delta = c \; a_{AA\mathcal{D}} \tag{3}$$

where A and g^2 imply a heavy operator $\mathcal{O}_A$ and the 't Hooft coupling repectively and $\mathcal{D}$ is the marginal operator with the conformal dimenion 4 in the four-dimensional $\mathcal{N}=4$ SYM. In the above, the factor c usually depends on the dimensionality of the dual gauge theory. In the four dimensional SYM theory, c is given by $2\pi^2$, which corresponds to the solid angle of S^3. There exists another interesting superconformal theory defined on $2+1$-dimension, the so-called ABJM model, which is described by a three-dimensional $\mathcal{N}=6$ Chern-Simons theory. The structure constant of the ABJM model was also expected to have the same relation like (3) [51]. In this section, we briefly review how we can derive the formula (3) of the dual conformal field theory [37].

We start with a conformal field theory defined on the d-dimensional Euclidean space, which is dual to a gravity theory on AdS_{d+1} space with the Euclidean signature following the AdS/CFT

correspondence

$$S_{CFT} = \frac{1}{g^2} \int d^d x \; \mathcal{L}_{CFT}, \tag{4}$$

where g^2 is the coupling constant. Then, the theory deformed by a marginal operator $\mathcal{D}$ with the conformal dimension d is given by

$$S_u = S_{CFT} + u \int d^d y \; \mathcal{D}(y), \tag{5}$$

where u is the dimensionless deformation parameter and the marginal operator $\mathcal{D}$ is the renormalized operator in the original conformal field theory. Now, consider any renormalized operator $\mathcal{O}_A$ of the undeformed theory and denote its conformal dimension by Δ_A. The correlation function of $\mathcal{O}_A$ with any other operators in the deformed theory is related to the correlation function of the undeformed theory by the following

$$\langle \mathcal{O}_A(x) \cdots \rangle_u = \langle \mathcal{O}_A(x) \cdots \rangle - u \int d^d y \left\langle \mathcal{O}_A(x) \mathcal{D}(y) \cdots \right\rangle, \tag{6}$$

where $\langle \cdots \rangle_u$ means a correlation function in the deformed theory. Notice that here we will regard only the first order correction of u. In the right hand side, when y approaches to x, there exists another new divergence determined by the operator product expansion (OPE)

$$\mathcal{D}(y) \mathcal{O}_A(x) = \sum_B \frac{a_{\mathcal{D}AB}}{|x-y|^{d+\Delta_A-\Delta_B}} \mathcal{O}_B(x), \tag{7}$$

where we assume that the complete basis of operators $\{\mathcal{O}_A\}$ is diagnonal with the unit norm

$$\langle \mathcal{O}_A(x) \mathcal{O}_B(0) \rangle = \frac{\delta_{AB}}{|x|^{2\Delta_A}}, \tag{8}$$

and $a_{\mathcal{D}AB}$ is exactly the structure constant of the 3-point function $\langle \mathcal{O}_A \mathcal{O}_B \mathcal{D} \rangle$ in the undeformed theory. The new divergence caused by the marginal deformation can be cancelled by renormalizing the operator $\mathcal{O}_A$ in the deformed theory. From now on, we concentrate on the case of $\Delta_A = \Delta_B$. In the integral of (6), the divergence arises at $y \to x$. By using the OPE in (7), we can find the log divergence

$$\int \frac{d^d y}{|x-y|^d} \approx - \frac{2\pi^{d/2}}{\Gamma(d/2)} \; \log |\epsilon| \tag{9}$$

where $\Delta_A = \Delta_B$ is used and $\frac{2\pi^{d/2}}{\Gamma(d/2)}$ and ϵ are the solid angle of the $(d-1)$-dimensional sphere S^{d-1} and an appropriate UV cut off respectively. By defining the renormalized operator $\mathcal{O}_A^u$ in the deformed theory

$$\mathcal{O}_A^u = \mathcal{O}_A - \frac{2\pi^{d/2}}{\Gamma(d/2)} \; u \sum_{\Delta_B = \Delta_A} a_{\mathcal{D}AB} \; \log |\epsilon| \; \mathcal{O}_B, \tag{10}$$

we can remove the divergence of the correlation function in (6). From the definition of the renormalized operator in the deformed theory, we can easily read off the two-point function

$$\langle \mathcal{O}^u_A(x)\mathcal{O}^u_B(0)\rangle = \frac{1}{|x|^{2\Delta_A}}\left[\delta_{AB} - \frac{2\pi^{d/2}}{\Gamma(d/2)}\; u\,(a_{\mathcal{D}AB} + a_{\mathcal{D}BA})\log|x|\right]. \tag{11}$$

Especially, for $\mathcal{O}_A = \mathcal{O}_B$, the above reduces to

$$\langle \mathcal{O}^u_A(x)\mathcal{O}^u_A(0)\rangle = \frac{1}{|x|^{2\Delta^u_A}}, \tag{12}$$

where the conformal dimension of the renormalized operator in the deformed theory is given by $\Delta^u_A = \Delta_A + \frac{2\pi^{d/2}}{\Gamma(d/2)}\; u\; a_{AA\mathcal{D}}$. As a result, we can easily see that the calculation of the conformal dimension of the renormalized operator in the marginally deformed theory provides information about the structure constant of the undeformed theory

$$\frac{\partial}{\partial u}\Delta^u_A = \frac{2\pi^{d/2}}{\Gamma(d/2)}\; a_{AA\mathcal{D}}. \tag{13}$$

If the marginal operator $\mathcal{D}$ is identified with the Lagrangian density operator of the conformal theory, the deformed theory is also the same conformal theory just with the modified gauge coupling g', which is related to the original gauge coupling at the linear order of u

$$g'^2 = g^2(1-u). \tag{14}$$

Using this relation, the derivative with respect to u in (13) can be rewritten, in terms of the derivative with respect to g^2, as

$$-\,g^2\frac{\partial}{\partial g^2}\Delta_A = \frac{2\pi^{d/2}}{\Gamma(d/2)}\; a_{AA\mathcal{D}}. \tag{15}$$

In the 5-dimensional AdS space ($d = 4$), the relation between the conformal dimension and the structure constant is simply reduced to

$$-\,g^2\frac{\partial}{\partial g^2}\Delta_A = 2\pi^2\; a_{AA\mathcal{D}}, \tag{16}$$

which has been proved by explicit calculation of the structure constant in various solitonic string configurations. For the ABJM model, the structure constant of two heavy operators and one marginal operator can be also related to the conformal dimension of the heavy operator by

$$-\,g^2\frac{\partial}{\partial g^2}\Delta_A = 4\pi\; a_{AA\mathcal{D}}. \tag{17}$$

In the followings, we will check this relation explicitly in various heavy operators of the ABJM model.

3 $AdS_4 \times CP^3$ as a dual geometry of the ABJM model

Now, we consider a string theory defined on $AdS_4 \times CP^3$. The CP^3 space can be described as the following. We first consider an 8-dimensional Euclidean space with real coordinates X_a $(a = 1, \cdots, 8)$. Imposing a constraint

$$\frac{R^2}{4} = \sum_{a=1}^{8} X_a^2, \tag{18}$$

the 8-dimensional Euclidean space reduces to S^7. In terms of complex coordinates $Z_i = X_i + iX_{i+4}$ $(i = 1, \cdots, 4)$, the above constraints can be rewritten as

$$\frac{R^2}{4} = \sum_{i=1}^{4} |Z_i|^2. \tag{19}$$

In order to go to a CP^3 space, we should impose one more constraint

$$0 = \sum_{i=1}^{4} (X_i \partial_\alpha X_{i+4} - X_{i+4} \partial_\alpha X_i) = \frac{i}{2} \sum_{i=1}^{4} \left(Z_i \partial_\alpha \bar{Z}_i - \bar{Z}_i \partial_\alpha Z_i \right), \tag{20}$$

where α implies coordinates of the string worldsheet. Due to these two constraints the resulting space becomes a 6-dimensional space. One of the coordinate parameterizations satisfying these two constraints can be given by

$$\begin{aligned}
Z_1 &= \frac{R}{2} \cos\xi \ \sin\theta \ e^{i\phi_1}, \\
Z_2 &= \frac{R}{2} \cos\xi \ \cos\theta \ e^{i\phi_2}, \\
Z_3 &= \frac{R}{2} \sin\xi \ \sin\theta \ e^{-i\phi_1}, \\
Z_3 &= \frac{R}{2} \sin\xi \ \cos\theta \ e^{-i\phi_2},
\end{aligned} \tag{21}$$

only for $\xi = \pi/4$. This coordinate parameterization describes the diagonal subspace S^3 of $S^3 \times S^3$ or RP^3 in CP^3 [31].

3.1 Point-like String in AdS_4

Let us consider a point particle moving on $AdS_4 \times CP^3$. The similar calculation $AdS_5 \times S^5$ has been already performed [37, 38, 39, 51]. Although all technical details for calculating two- and three-point correlation functions are almost the same as the AdS_5 case, it is still interesting to check the AdS/CFT correspondence in (13) with the exact results of the correlation functions. In this section,

we will summeraize the results without explaining the details (see the details [37, 48, 51, 52, 53]). The action describing a particle propagating on AdS_4 only is given by the Polyakov-type action

$$S_P[X, s, \Phi = 0] = \frac{1}{2} \int_{-s/2}^{s/2} d\tau \left(\frac{\dot{x}^i \dot{x}_i + \dot{z}^2}{z^2} - m^2 \right) , \tag{22}$$

where x^i and z are coordinates of AdS_4 and we use the Euclidean AdS_4 metric. The solutions satisfying the equation of motion becomes

$$\begin{aligned} x(\tau) &= R \tanh \kappa\tau + x_0, \\ z(\tau) &= \frac{R}{\cosh \kappa\tau}, \end{aligned} \tag{23}$$

where R corresponds to the AdS radius. Under the following boundary conditions with an appropriate UV cutoff ϵ

$$\{x(-s/2), z(-s/2)\} = \{x_i, \epsilon\} \quad \text{and} \quad \{x(s/2), z(s/2)\} = \{x_f, \epsilon\} , \tag{24}$$

we can easily find relations between parameters

$$\kappa \approx \frac{2}{s} \log \frac{x_f}{\epsilon} \quad \text{and} \quad x_f \approx 2R \approx 2x_0. \tag{25}$$

For evaluating the semiclassical partition function of this point particle, first we should find a saddle point of the on-shell action

$$S_P[X, s, \Phi] = \frac{2}{s} \log^2 \frac{x_f}{\epsilon} - \frac{1}{2} m^2 s . \tag{26}$$

Then, the saddle point in terms of the modular parameter $\bar{s}$ is given by

$$\bar{s} = -i \frac{2}{m} \log \frac{x_f}{\epsilon} . \tag{27}$$

Finally, the semiclassical partition function becomes

$$e^{iS_P[\bar{X}, \bar{s}, \Phi]} = \left(\frac{\epsilon}{x_f} \right)^{2\Delta} \quad \text{with} \quad \Delta = m , \tag{28}$$

which, according to the AdS/CFT correspondence, represents the two-point correlation function of the dual operator to a point particle. Notice that the above result is the exactly same as the AdS_5 case.

Now, we consider a three-point correlation function including a marginal operator. Although two point functions on AdS_5 and on AdS_4 are same, the three point function should be different due to the different dilaton field propagator. In the AdS_4 space, the boundary-bulk propagator of a massless scalar field (dilaton) is given by

$$\mathcal{D}_4 = \frac{4}{\pi^2} \left(\frac{z}{z^2 + (x - y)^2} \right)^3 . \tag{29}$$

Using this propagator, the three point correlation function between two heavy point particles and one massless scalar operator is described by

$$\langle \mathcal{O}_A(x_f)\mathcal{O}_A(0)\mathcal{D}(y)\rangle = \frac{I}{x_f^{2\Delta}}, \tag{30}$$

where Δ is the conformal dimension of the point particle and I is given by

$$\begin{aligned} I[X,s,y] &= i\frac{1}{2\pi^2}\int_{-s/2}^{s/2} d\tau \left(\frac{\dot{x}^i\dot{x}_i+\dot{z}^2}{z^2}-m^2\right)\left(\frac{z}{z^2+(x-y)^2}\right)^3 & (31)\\ &= -\frac{m}{16\pi}\frac{x_f^3}{(x_f-y)^3y^3} & (32)\end{aligned}$$

As a result, the three point correlation function of two massive point particles and a massless scalar field becomes

$$\langle \mathcal{O}_A(x_f)\mathcal{O}_A(0)\mathcal{D}(y)\rangle = -\frac{m}{16\pi}\frac{1}{x_f^{2\Delta-3}|x_f-y|^3y^3}, \tag{33}$$

Notice that the coordinate dependence of this three point functions is consistent with the expectation of the conformal theory, where it is fully determined by the global conformal symmetry. Assuming that $\Delta = m \sim \sqrt{g}$ [37], we can easily check that the structure constant in the above coincides with the results obtained by the previous RG analysis [51]

$$-g^2\frac{\partial\Delta}{\partial g^2} = 4\pi a_{AA\mathcal{D}} = -\frac{m}{4}. \tag{34}$$

3.2 A circular string wrapped in θ

Now, consider a circular string rotating on S^3, which is a diagonal subgroup of $S^3\times S^3 \subset CP^3$. In order to describe this geometry, we take the previous parameterization in (21). In terms of real coordinates, the S^3 metric is given by

$$ds^2 = \frac{1}{4}R^2\left[d\theta^2+d\xi^2+\sin^2\theta d\phi_1^2+\cos^2\theta d\phi_2^2\right]. \tag{35}$$

Under the following circular string ansatz

$$\theta=\sigma\,,\ \phi_1=\omega_1\tau\,,\ \phi_2=\omega_2\tau\,, \tag{36}$$

which describes a string extended in θ with rotations in ϕ_1 and ϕ_2, the Polyakov string action becomes

$$S=\frac{T}{2}\int_{-s/2}^{s/2}d\tau\int_0^{2\pi}d\sigma\left[\frac{\dot{x}^i\dot{x}_i+\dot{z}^2}{z^2}-\theta'^2+\sin^2\theta\ \dot{\phi}_1^2+\cos^2\theta\ \dot{\phi}_2^2\right], \tag{37}$$

where dot and prime mean the derivative with respect to τ and σ respectively and the string tension T in $AdS_4 \times CP^3$ is given by [24, 26, 25]

$$T = 2g = \sqrt{\frac{\lambda}{2}}, \tag{38}$$

where λ is the 't Hooft coupling constant. Following Ref. [38, 37] the total action, after considering the convolution with the relevant wave function, becomes

$$\begin{aligned} \bar{S} &\equiv S - \Pi_{\phi_1}\dot{\phi}_1 - \Pi_{\phi_2}\dot{\phi}_2 && (39)\\ &= \frac{1}{2}\pi s T(-2 + 2\kappa^2 - \omega_1^2 - \omega_2^2), && (40) \end{aligned}$$

in which S implies the original action including both the AdS_4 and S^3 parts. Using (25), the saddle point is located at

$$\bar{s} = \frac{2\sqrt{2}}{i\sqrt{2+\omega_1^2+\omega_2^2}} \log\left(\frac{x_f}{\epsilon}\right). \tag{41}$$

Then, the semiclassical partition function, which describes the two point function of the dual operator to the circular string, becomes

$$e^{i\bar{S}} = \left(\frac{\epsilon}{x_f}\right)^{2\Delta} \tag{42}$$

with

$$\Delta = \sqrt{2}\pi T\sqrt{2+\omega_1^2+\omega_2^2}, \tag{43}$$

which corresponds to the conformal dimension of the dual operator. In terms of the angular momenta J_1 and J_2

$$J_1 = \pi T\omega_1 \quad \text{and} \quad J_2 = \pi T\omega_2, \tag{44}$$

the conformal dimension can be rewritten as

$$\Delta = \sqrt{2}\ \sqrt{J_1^2 + J_2^2 + 2\pi^2 T^2}. \tag{45}$$

Then, from (17) we can expect that the structure constant of the three point function between two circular strings and one marginal scalar operator is given by

$$4\pi\ a_{AA\mathcal{D}} = -\frac{\sqrt{2}\pi^2 T^2}{\sqrt{J_1^2+J_2^2+2\pi^2T^2}}. \tag{46}$$

Now, we check the this structure constant by the explicit calculation in the string theory. In the strong 't Hooft coupling regime, the three point correlation function of two heavy $\mathcal{O}_A$ and one marginal $\mathcal{D}$ operators can be described by

$$\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{D}(y)\rangle \ \approx\ \frac{I}{x_f^{2\Delta}}, \tag{47}$$

where the denominator corresponds to the two point function of $\mathcal{O}_A$ in (42) and I is the multiplication of the action and the propagator of bulk scalar fluctuation corresponding to the marginal operator in (29)

$$I = i\frac{T}{\pi^2}\int_{-s/2}^{s/2} d\tau \int_0^{2\pi} d\sigma \left(\frac{\dot{x}^i\dot{x}_i + \dot{z}^2}{z^2} - \theta'^2 + \sin^2\theta\ \dot{\phi}_1^2 + \cos^2\theta\ \dot{\phi}_2^2\right)\left(\frac{z}{z^2+(x-y)^2}\right)^3 \tag{48}$$

$$= -\frac{\sqrt{2}\,T}{4\sqrt{2+\omega_1^2+\omega_2^2}}\ \frac{1}{x_f^{-3}(x_f-y)^3 y^3}, \tag{49}$$

In terms of the angular momenta, the three point correlation function becomes

$$\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{D}(y)\rangle = -\frac{\sqrt{2}\pi T^2}{4\sqrt{J_1^2+J_2^2+2\pi^2T^2}}\ \frac{1}{x_f^{2\Delta-3}|x_f-y|^3 y^3}, \tag{50}$$

in which we can see that the structure constant is consistent with the expectation of the RG analysis in (46) [51].

3.3 A circular string wrapped in ϕ_1

Let us consider another circular string which is wrapped in ϕ_1 and rotating in ϕ_1 and ϕ_2. The appropriate ansatz for it is given by

$$\phi_1 = \omega_1\tau + w\sigma \quad , \quad \phi_2 = \omega_2\tau \quad \text{and} \quad \theta = \theta_0, \tag{51}$$

where w is the winding number and $0 \le \sigma < 2\pi$ Assume that the position of the string in θ is fixed to θ_0 and that ω_1 and ω_2 are finite. In such parameterization, $\theta_0 = \pi/2$ corresponds to a equator of S^2 described by ϕ_1 and ϕ_2. So, at a fixed τ and for $w = 1$ the above ansatz describe the string wrapping the equator of S^2 once.

From the general string action

$$S = \frac{T}{2}\int_{-s/2}^{s/2} d\tau \int_0^{2\pi} d\sigma \Bigg[\frac{\left(\dot{x^i}\right)^2 - \left(x^{i\prime}\right)^2 + \dot{z}^2 - z'^2}{z^2} + \dot{\theta}^2 - \theta'^2 + \sin^2\theta\left(\dot{\phi}_1^2 - \phi_1'^2\right) + \cos^2\theta\left(\dot{\phi}_2^2 - \phi_2'^2\right)\Bigg], \tag{52}$$

after inserting the convolution (39) and setting $\sin\theta_0 = \sqrt{1-\delta^2}$ with $\delta << 1$, the total action reduces to

$$\bar{S} = \pi s T\ \left[\kappa^2 - (w^2+\omega_1^2)(1-\delta^2) - \delta^2\omega_2^2\right], \tag{53}$$

which together with (25), shows that the saddle point is located at

$$\bar{s} = \frac{2}{i\sqrt{(w^2+\omega_1^2)(1-\delta^2)+\omega_2^2\,\delta^2}}\log\frac{x_f}{\epsilon}. \tag{54}$$

Then, the semiclassical partition function corresponding to the boundary two point function reads off

$$e^{i\bar{S}} = \left(\frac{\epsilon}{x_f}\right)^{2\Delta}, \tag{55}$$

with the following conformal dimension in terms of the two angular momenta J_1 and J_2 and the winding number w

$$\begin{aligned} \Delta &= 2\pi T\sqrt{(w^2+\omega_1^2)(1-\delta^2)+\omega_2^2\,\delta^2} \\ &= 2\pi T\sqrt{\frac{J_1^2}{4\pi^2T^2(1-\delta^2)}+(1-\delta^2)w^2+\frac{J_2^2}{4\pi^2T^2\delta^2}}, \end{aligned} \tag{56}$$

where the angular momenta are given by

$$\begin{aligned} J_1 &= 2\pi T\omega_1(1-\delta^2), \\ J_2 &= 2\pi T\omega_2\delta^2. \end{aligned} \tag{57}$$

In order to understand this result in depth, we first consider the $\delta = 0$ case, in which since S^3 reduces to S^2, the conformal dimension of the circular string wrapped in ϕ_1 is also reduced to that defined on S^2

$$\bar{\Delta} = \sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2}, \tag{58}$$

where $\bar{\Delta}$ and $\bar{J}_1 \equiv 2\pi T\omega_1$ imply the values of Δ and J_1 at $\delta = 0$ or S^2. In the large 't Hooft coupling limit ($T >> 1$), since ω_1 and ω_2 are finite, Δ and J_1 are large ($\sim T$) but J_2 is proportional to $T\delta^2$. If we define $\bar{J}_2 \equiv 2\pi T\omega_2$, all values with the bar symbol are finite and proportional to T. Then, the conformal dimension in (56) can be rewritten in terms of the bar variables as

$$\Delta = \sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2 + \left(\bar{J}_2^2 - \bar{J}_1^2 - 4\pi^2T^2w^2\right)\delta^2}, \tag{59}$$

which is useful to understand the size effect of the circular string wrapped in ϕ_1. Near the equator of S^2 ($\delta << 1$) with the large 't Hooft coupling, the conformal dimension of this circular string can be expanded to

$$\Delta = \sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2} + \frac{\bar{J}_2^2 - \bar{J}_1^2 - 4\pi^2T^2w^2}{2\sqrt{\bar{J}_1^2 + 4\pi^2T^2w^2}}\,\delta^2 + \cdots, \tag{60}$$

where the ellipsis implies the higher order corrections. In the above, the first term is nothing but the conformal dimension of the circular string lying at the equator of S^2 and the second implies the size effect due to the change of the string length. If the circular string lies near the equator of S^2 which implies that $\sin\theta_0 = \sqrt{1-\delta^2}$ with $\delta << 1$, the string length l is given by

$$l = 2\pi\sqrt{1-\delta^2} \approx 2\pi - \pi\delta^2, \tag{61}$$

where $\pi\delta^2$ is the change of the string length from one lying at equator. Therefore, the second term of the conformal dimension in (60) corresponds to the size effect. Rewriting δ^2 in terms of J_1 and $\bar{J}_1$

$$\delta^2 = \frac{\bar{J}_1 - J_1}{\bar{J}_1}, \tag{62}$$

the conformal dimension of the circular string wrapped in ϕ_1 becomes

$$\Delta = \sqrt{\bar{J}_1^2 + 4\pi^2 T^2 w^2 + \left(\bar{J}_2^2 - \bar{J}_1^2 - 4\pi^2 T^2 w^2\right) \frac{\bar{J}_1 - J_1}{\bar{J}_1}}. \tag{63}$$

This is similar to the finite size effect of the magnon or the spike. More precisely, since the dual operator of the circular string is the closed spin chain, it can describe the size effect of the closed spin chain. For the circular string, the size effect is suppressed by the order of $(\bar{J}_1 - J_1)$ when $J_1 \to \bar{J}_1$, whereas the finite size effect of magnon corresponding to the open spin chain is exponentially suppressed with J_1, $\Delta_{size} \sim e^{-J_1}$.

Now, let us consider the three point function with two circular strings and one massless dilaton. Following the previous methods, we can find that the three point function is given by

$$\langle \mathcal{O}_A(0) \mathcal{O}_A(x_f) \mathcal{D}(y) \rangle \ = \frac{a_{AA\mathcal{D}}}{x_f^{2\Delta - 3} |x_f - y|^3 y^3} \tag{64}$$

with the following structure constant

$$a_{AA\mathcal{D}} = -\frac{\pi T^2 \ w^2 \ J_1}{2\bar{J}_1 \sqrt{\bar{J}_1^2 + 4\pi^2 T^2 w^2 + \left(\bar{J}_2^2 - \bar{J}_1^2 - 4\pi^2 T^2 w^2\right) \frac{\bar{J}_1 - J_1}{\bar{J}_1}}}, \tag{65}$$

which is consistent with the result obtained from the RG analysis, $a_{AA\mathcal{D}} = -\frac{T}{8\pi} \frac{\partial \Delta}{\partial T}$ in (17). In the large 't Hooft coupling limit, the structure constant can be expanded up to $\mathcal{O}(\delta^2)$ to

$$a_{AA\mathcal{D}} \approx -\frac{\pi T^2 w^2}{2\sqrt{\bar{J}_1^2 + 4\pi^2 T^2 w^2}} + \frac{\pi T^2 w^2 (\bar{J}_1^2 + \bar{J}_2^2 + 4\pi^2 T^2 w^2)}{4\left(\bar{J}_1^2 + 4\pi^2 T^2 w^2\right)^{3/2}} \ \frac{\bar{J}_1 - J_1}{\bar{J}_1}, \tag{66}$$

in which the first is the structure constant for the circular string defined on the equator of S^2 and the second is the size corrections due to the change of the string length respectively.

3.4 Dyonic Magnon

Now, we consider more nontrivial solution called a dyonic magnon solution. In the $AdS_5 \times S^5$ background dual to the $N = 4$ Super Yang-Mills, the conformal dimension and the three point function of the dyonic magnon have been investigated [6, 7, 8, 52, 54, 55, 56]. In the $AdS_4 \times CP^3$, since the similar structure of the dyonic magnon to one in $AdS_5 \times S^5$, we can easily expect the similar results

[48, 52, 53]. But due to the different string tension (or the coupling constant) and the propagator of the scalar field, the final results are slightly different from the $AdS_5 \times S^5$ case. Here, we will summarize the results without explaining the details (for the details, see [48, 52, 53]) and check the RG analysis in (17).

The dyonic magnon solution corresponds to the bound state of magnons in the spin chain model, which can be described by a solitonic string rotating on $R \times S^3$ where R is the time in AdS_4 and S^3 is the subspace of CP^3. The ansatz for the dyonic string is given by

$$\theta = \theta(y) \quad , \quad \phi_1 = \nu_1 \tau + g_1(y) \quad \text{and} \quad \phi_2 = \nu_2 \tau + g_2(y), \tag{67}$$

with

$$y = a\tau + b\sigma, \tag{68}$$

where $g_1(y)$ and $g_2(y)$ are arbitrary functions. Due to the rotational symmetries in ϕ_1 and ϕ_2, $g_1(y)$ and $g_2(y)$ should satisfy the following first order differential equations

$$\begin{aligned} g_1' &= \frac{1}{b^2 - a^2} \left(a\nu_1 - \frac{c_1}{\sin^2\theta} \right), \\ g_2' &= \frac{1}{b^2 - a^2} \left(a\nu_2 - \frac{c_2}{\cos^2\theta} \right), \end{aligned} \tag{69}$$

where c_1 and c_2 are integration constants and the prime means the derivative with respect to y. Here, we assume that $b^2 > a^2$ which describes the dyonic magnon and concentrate on this case. The different parameter range $a^2 > b^2$ explains the dyonic spike [24, 48, 52]. In the infinite size limit of the dyonic magnon lying at $\theta_{max} = \frac{\pi}{2}$, usually the energy and the first angular momentum in ϕ_1-direction become infinite but the second angular momentum in ϕ_2 is finite, so we should take $c_2 = 0$ to make the second angular momentum finite. Using this fact, the equation for θ can be rewritten as

$$\theta'^2 = \frac{b^2(\nu_1^2 - \nu_2^2)}{(b^2 - a^2)^2 \sin^2\theta} \left(\sin^2\theta_{max} - \sin^2\theta \right) \left(\sin^2\theta - \sin^2\theta_{min} \right), \tag{70}$$

with

$$\sin^2\theta_{max} = \frac{c_1}{a\nu_1}, \tag{71}$$

$$\sin^2\theta_{min} = \frac{a\nu_1 c_1}{b^2(\nu_1^2 - \nu_2^2)}. \tag{72}$$

Here, θ_{max} implies the position for two ends of the solitonic string whereas θ_{min} corresponds to the position for the center of it. To investigate the dyonic magnon solution, we should further impose another boundary condition at θ_{max} determined by $\theta'_{max} = 0$ [48, 52, 53]. The appropriate boundary condition for the dyonic magnon is given by $\partial_\sigma \phi_1 = 0$ at θ_{max}, which gives rise to (71). If we choose

a general value for c_1, we can calculate the finite size effect of the conformal dimension and the three point function. For the giant magnon and the dyonic magnon in the $AdS_5 \times S^5$, the finite size effect have been calculated in [53, 54]. From now on, we will concentrate only on the infinite size limit just for checking the consistency of the RG analysis, in which $\sin\theta_{max} = 1$ and $c_1 = a\nu_1$.

Now, we evaluate the semiclassical partition function of the dyonic magnon following the method used in the previous section. After the convolution with the relevant wave function, the total action of the dyonic magnon becomes

$$\begin{aligned}\bar{S} &\equiv S - \Pi_{\phi_1}\dot{\phi}_1 - \Pi_{\phi_2}\dot{\phi}_2 \\ &= (\kappa^2 - \nu_1^2) sLT,\end{aligned} \tag{73}$$

where $2L$ is the length of the worldsheet string, $2L \equiv \int_{-L}^{L} d\sigma$. Then, the saddle point is given by

$$\bar{s} = -i\frac{2}{\nu_1}\log\frac{x_f}{\epsilon}, \tag{74}$$

At the saddle point, the semiclassical partition function reduces to

$$e^{iS_{tot}} = \left(\frac{\epsilon}{x_f}\right)^{2\Delta}, \tag{75}$$

with the following conformal dimension Δ

$$\Delta = J_1 + \sqrt{J_2^2 + 4T^2\sin^2\frac{p}{2}}, \tag{76}$$

where two angular momenta, J_1 and J_2, and the worldsheet momentum p are given by [52]

$$J_1 = \frac{2T\nu_1}{\sqrt{\nu_1^2-\nu_2^2}}\int_{\theta_{min}}^{\pi/2} d\theta\,\frac{\sin\theta}{\cos\theta}\frac{1}{\sqrt{\sin^2\theta - \sin^2\theta_{min}}}\left(\sin^2\theta - \frac{\nu_1^2-\nu_2^2}{\nu_1^2}\sin^2\theta_{min}\right), \tag{77}$$

$$J_2 = \frac{2T\nu_2}{\sqrt{\nu_1^2-\nu_2^2}}\int_{\theta_{min}}^{\pi/2} d\theta\,\frac{\sin\theta\cos\theta}{\sqrt{\sin^2\theta - \sin^2\theta_{min}}} = \frac{2T\nu_2\cos\theta_{min}}{\sqrt{\nu_1^2-\nu_2^2}}, \tag{78}$$

$$\frac{p}{2} = \int_{\theta_{min}}^{\pi/2} d\theta\,\frac{\sin\theta_{min}\cos\theta}{\sin\theta\sqrt{\sin^2\theta - \sin^2\theta_{min}}} = \arcsin\left(\cos\theta_{min}\right). \tag{79}$$

The three point function between two dyonic magnons and a marginal scalar operator can be also easily evaluated. After some calculations, the three point function is given by

$$\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{D}(y)\rangle = -\frac{T\sqrt{\nu_1^2-\nu_2^2}\cos\theta_{min}}{4\pi\nu_1}\frac{1}{x_f^{2\Delta-3}|x_f-y|^3y^3}, \tag{80}$$

where we use $\kappa = i\nu$. Using the results in (78) and (79), the structure constant in terms of J_2 and p reduces to

$$a_{AA\mathcal{D}} = -\frac{T^2\sin^2(p/2)}{2\pi\sqrt{J_2^2 + 4T^2\sin^2(p/2)}}, \tag{81}$$

which is consistent with the result of the RG analysis

$$a_{AA\mathcal{D}} = -\frac{g^2}{4\pi}\frac{\partial}{\partial g^2}\Delta = -\frac{T^2\sin^2(p/2)}{2\pi\sqrt{J_2^2+4T^2\sin^2(p/2)}}. \tag{82}$$

4 Three point function with an (ir)relevant scalar operator

In the previous section, we calculated the conformal dimensions of various heavy operators and the three point functions with a marginal scalar operator at the large 't Hooft coupling limit. Furthermore, we also checked that those results are exactly matched with the expectations of the RG analysis in the dual gauge theory. What is the three point function with the (ir)relevant scalar operator instead of the marginal one? In such case, since the light operator is (ir)relevant, we can not apply the RG analysis to determined the structure constant and moreover the perturbative method in the dual gauge theory is not also applicable. However, the AdS/CFT correspondence can provide the answer, so it is very interesting to calculate such three point function.

In order to consider an (ir)relevant scalar operator, we introduce a massive dilaton field which propagates in the $d+1$-dimensional AdS space. The marginal operator studied previously is the special case of the (ir)relevant one and it corresponds to the massless dialton. According to the AdS/CFT correspondence, the massive dilaton with mass m_ϕ corresponds to the scalar operator with conformal dimensions h_+

$$h_\pm = \frac{d}{2} \pm \frac{\sqrt{d^2+4m_\phi^2}}{2}, \tag{83}$$

where h_- usually represents the conformal dimension of the source in the dual theory. From now on, we set $h_+ = h$ for simplicity. Noting that it is allowed to have negative mass square in the AdS space, so $m^2 < 0$ or $m^2 > 0$ correspond to a relevant or an irrelevant operator respectively. The bulk-boundary propagator of this massive dilaton in the $d+1$-dimensional AdS space is given by [57, 58, 59]

$$\mathcal{D}_m = \frac{\Gamma(h)}{\pi^{d/2}\,\Gamma(h-d/2)}\frac{z^h}{(z^2+(x-y)^2)^h}. \tag{84}$$

Using this result we can easily evaluate the three point function between two heavy and one (ir)relevant light operators

$$\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{D}(y)\rangle_{m_\phi} \approx \frac{I}{x_f^{2\Delta}} \tag{85}$$

with

$$I = i\frac{T}{4\gamma}\int_{-s/2}^{s/2} d\tau \int d\sigma\ \mathcal{L}\ \mathcal{D}_m, \tag{86}$$

where Δ and $\mathcal{L}$ are the conformal dimension and the Lagrangian of the solitonic string and γ is either 1 for the solitonic string or 2 for the point-like string due to the different dilaton coupling of the action. From now on, we concentrate on the 4-dimensional AdS space. It can be easily generalized to other dimensions. In the following, we summarize the results for the three point function with an (ir)relevant operator.

i) For a point-like String in AdS_4

$$\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{D}(y)\rangle \;=-\frac{m}{2^{h+2}\pi}\frac{\Gamma\left(\frac{h}{2}\right)\Gamma\left(h\right)}{\Gamma\left(\frac{h+1}{2}\right)\Gamma\left(h-\frac{3}{2}\right)}\;\frac{1}{x_f^{2\Delta-h}\;|x_f-y|^h\;y^h}, \tag{87}$$

where m is the mass of the point particle related to the coupling, $m \sim \sqrt{g}$.

ii) For a circular string wrapped in θ

$$\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{D}(y)\rangle \;=-\frac{\pi T^2}{2^{h-1/2}\sqrt{J_1^2+J_2^2+2\pi^2T^2}}\frac{\Gamma\left(\frac{h}{2}\right)\Gamma\left(h\right)}{\Gamma\left(\frac{h+1}{2}\right)\Gamma\left(h-\frac{3}{2}\right)}\;\frac{1}{x_f^{2\Delta-h}\;|x_f-y|^h\;y^h}. \tag{88}$$

iii) For a circular string wrapped in ϕ_1

$$\begin{aligned}\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{D}(y)\rangle \;&=\; -\frac{\pi T^2\;w^2\;J_1}{2^{h-1}\bar{J}_1\sqrt{\bar{J}_1^2+4\pi^2T^2w^2+\left(\bar{J}_2^2-\bar{J}_1^2-4\pi^2T^2w^2\right)\frac{\bar{J}_1-J_1}{\bar{J}_1}}}\\ &\quad\times\frac{\Gamma\left(\frac{h}{2}\right)\Gamma\left(h\right)}{\Gamma\left(\frac{h+1}{2}\right)\Gamma\left(h-\frac{3}{2}\right)}\;\frac{1}{x_f^{2\Delta-h}|x_f-y|^hy^h},\end{aligned} \tag{89}$$

where w means the winding number of a circular string in the ϕ_1-direction and we use the previous definitions, $\bar{J}_1=2\pi T\omega_1$, $\bar{J}_2=2\pi T\omega_2$ and $J_1=2\pi T\omega_1\sin^2\theta_0$.

iv) For a dyonic magnon

$$\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{D}(y)\rangle \;=-\frac{T^2\sin^2(p/2)}{2^{h-1}\pi\sqrt{J_2^2+4T^2\sin^2(p/2)}}\frac{\Gamma\left(\frac{h}{2}\right)\Gamma\left(h\right)}{\Gamma\left(\frac{h+1}{2}\right)\Gamma\left(h-\frac{3}{2}\right)}\;\frac{1}{x_f^{2\Delta-h}\;|x_f-y|^h\;y^h}. \tag{90}$$

Interestingly, the results of the solitonic string except the point-like string show that the three point function with an (ir)relevant scalar operator is related to that with a marginal scalar operator.

If we denote the structure constant of the three point function with an (ir)relevant scalar operator as $a_{AA\mathcal{D}_m}$ and that with a marginal scalar operator as $a_{AA\mathcal{D}_0}$, then $a_{AA\mathcal{D}_m}$ can be rewritten as

$$a_{AA\mathcal{D}_m} = \frac{1}{2^{h-2}} \frac{\Gamma\left(\frac{h}{2}\right)\Gamma\left(h\right)}{\Gamma\left(\frac{h+1}{2}\right)\Gamma\left(h-\frac{3}{2}\right)} a_{AA\mathcal{D}_0}, \tag{91}$$

where the extra factor $\frac{1}{2^{h-2}} \frac{\Gamma\left(\frac{h}{2}\right)\Gamma(h)}{\Gamma\left(\frac{h+1}{2}\right)\Gamma\left(h-\frac{3}{2}\right)}$ is originated from the integration of the bulk to boundary propagator for the massive dilaton. The position dependence of the above three point functions have the usual form expected by the global conformal symmetry. As a result, the three point function between two extended strings and an (ir)relevant operator is given by

$$\langle \mathcal{O}_A(0)\mathcal{O}_A(x_f)\mathcal{D}(y)\rangle_{m_\phi} = \frac{1}{2^{h-2}} \frac{\Gamma\left(\frac{h}{2}\right)\Gamma\left(h\right)}{\Gamma\left(\frac{h+1}{2}\right)\Gamma\left(h-\frac{3}{2}\right)} \; \frac{a_{AA\mathcal{D}_0}}{x_f^{2\Delta-h}\; |x_f-y|^h \; y^h}, \tag{92}$$

which can reproduce the results of the three point functions with a marginal operator by taking $m_\phi = 0$. These results are also easily generalized to the higher dimensional case like $AdS_5 \times S^5$, in which the definition of the string tension is modified [26].

5 Discussion

In this paper, we have checked the AdS/CFT correspondence by using the various solitonic string solutions. First, we considered solutions for a point-like, two circular strings and dyonic magnon rotating in the diagonal S^3 of CP^3. By evaluating the semiclassical partition functions, we calculated the energies of them, which correspond to the conformal dimension of the dual heavy operators defined on the dual gauge theory. After this, we also calculated the three point correlation functions between those two heavy operators and one marginal scalar operator, which is dual to the massless dilaton in the bulk. Following the AdS/CFT correspondence, these three point function should be also related to the results of the gauge theory. However, calculating the general three point function in the large 't Hooft coupling limit is almost impossible because we have no technique to manipulate the theory in the strong coupling regime. But interestingly there are several exceptions. One of them is that if the dual gauge theory is conformal and this theory is deformed by a marginal Lagrangian density operator, we can calculate the three point function by using the RG analysis even in the strong coupling regime. In section 2. we summarized the relationship between the conformal dimension of the heavy operator and the structure constant of the three point function in the general dimensional conformal gauge theory following [37], in which the details of the gauge theory is not important. In order to check the AdS/CFT correspondence, we also evaluated the three point function with a massless dilaton dual to a marginal scalar operators in the dual gravity to the ABJM model. In the string theory, it is possible to calculate the three point functions directly in the large 't Hooft coupling regime. The results of

our work shows that the three point functions calculated in the string theory and in the dual gauge theory are perfectly matched, so the AdS/CFT correspondence is working very well. In the middle of this calculation, we desribed a very interesting solitonic string wrapped in ϕ_1, which is dual to a certain closed spin chain of the dual theory and gives information about the size effect of the closed spin chain. In the large 't Hooft coupling limit, it was shown that the size effect of the closed spin chain is suppressed by the order of $(\bar{J}_1 - J_1)$ as $J_1 \to \bar{J}_1$, which is totally different from the finite size effect of the open spin chain.

If we replace a marginal operator by an (ir)relevant one, can we calculate the three point function? In the gauge theory side it is impossible because the RG analysis is not working any more, so we can not calculate the three point function at least analytically. However, in the string side it is possible if we regard the large 't Hooft coupling limit and the conformal dimension of an (ir)relevant operator is very smaller than that of the heavy operator. The reason is that in the large 't Hooft coupling limit it is possible to expand perturbatively the string action in terms of $\sqrt{\lambda}$. Using this fact, we also evaluated the three point function with an (ir)relevant operator. Interestingly, the resulting form shows that the structure constant of the three point function with an (ir)relevant operator is closely related to that with a marginal operator and the coordinate dependence is given by the form expected by the conformal symmetry.

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References

[1] J. A. Minahan and K. Zarembo, JHEP **0303**, 013 (2003) [arXiv:hep-th/0212208].

[2] N. Beisert, J. A. Minahan, M. Staudacher and K. Zarembo, JHEP **0309**, 010 (2003) [arXiv:hep-th/0306139].

[3] N. Beisert and M. Staudacher, Nucl. Phys. B **670**, 439 (2003) [arXiv:hep-th/0307042].

[4] M. Grigoriev and A. A. Tseytlin, Nucl. Phys. B **800**, 450 (2008) [arXiv:0711.0155 [hep-th]].

[5] D. M. Hofman and J. M. Maldacena, J. Phys. A **39**, 13095 (2006) [hep-th/0604135].

[6] N. Dorey, J. Phys. A **39**, 13119 (2006) [hep-th/0604175].

[7] H. -Y. Chen, N. Dorey and K. Okamura, JHEP **0609**, 024 (2006) [hep-th/0605155].

[8] R. Ishizeki and M. Kruczenski, Phys. Rev. D **76**, 126006 (2007) [arXiv:0705.2429 [hep-th]].

[9] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Nucl. Phys. B **636**, 99 (2002) [hep-th/0204051].

[10] A. A. Tseytlin, hep-th/0409296.

[11] S. A. Frolov, R. Roiban and A. A. Tseytlin, Nucl. Phys. B **731**, 1 (2005) [hep-th/0507021].

[12] G. Arutyunov, S. Frolov and M. Zamaklar, Nucl. Phys. B **778**, 1 (2007) [hep-th/0606126].

[13] B. -H. Lee, R. R. Nayak, K. L. Panigrahi and C. Park, JHEP **0806**, 065 (2008) [arXiv:0804.2923 [hep-th]].

[14] J. Bagger and N. Lambert, Phys. Rev. D **77**, 065008 (2008) [arXiv:0711.0955 [hep-th]].

[15] A. Gustavsson, Nucl. Phys. B **811**, 66 (2009) [arXiv:0709.1260 [hep-th]].

[16] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, JHEP **0810**, 091 (2008) [arXiv:0806.1218 [hep-th]].

[17] G. Grignani, T. Harmark and M. Orselli, Nucl. Phys. B **810**, 115 (2009) [arXiv:0806.4959 [hep-th]].

[18] G. Grignani, T. Harmark, M. Orselli and G. W. Semenoff, JHEP **0812**, 008 (2008) [arXiv:0807.0205 [hep-th]].

[19] D. Astolfi, V. G. M. Puletti, G. Grignani, T. Harmark and M. Orselli, Nucl. Phys. B **810**, 150 (2009) [arXiv:0807.1527 [hep-th]].

[20] D. Astolfi, V. G. M. Puletti, G. Grignani, T. Harmark and M. Orselli, JHEP **1004**, 079 (2010) [arXiv:0912.2257 [hep-th]].

[21] I. Shenderovich, arXiv:0807.2861 [hep-th].

[22] T. McLoughlin and R. Roiban, JHEP **0812**, 101 (2008) [arXiv:0807.3965 [hep-th]].

[23] L. F. Alday, G. Arutyunov and D. Bykov, JHEP **0811**, 089 (2008) [arXiv:0807.4400 [hep-th]].

[24] B. H. Lee, K. L. Panigrahi and C. Park, JHEP **0811**, 066 (2008) [arXiv:0807.2559 [hep-th]].

[25] B. Gwak, B. -H. Lee, K. L. Panigrahi and C. Park, JHEP **0904**, 071 (2009) [arXiv:0901.2795 [hep-th]].

[26] B. H. Lee and C. Park, J. Korean Phys.Soc. **57** 30 (2010) [arXiv:0812.2727 [hep-th]].

[27] C. Kalousios, M. Spradlin and A. Volovich, JHEP **0907**, 006 (2009) [arXiv:0902.3179 [hep-th]].

[28] C. Ahn and R. I. Nepomechie, JHEP **0809**, 010 (2008) [arXiv:0807.1924 [hep-th]].

[29] A. Jevicki and K. Jin, JHEP **0906**, 064 (2009) [arXiv:0903.3389 [hep-th]].

[30] J. A. Minahan, W. Schulgin and K. Zarembo, JHEP **0903** (2009) 057 [arXiv:0901.1142 [hep-th]].

[31] M. C. Abbott, I. Aniceto and O. Ohlsson Sax, Phys. Rev. D **80**, 026005 (2009) [arXiv:0903.3365 [hep-th]].

[32] D. Bombardelli, D. Fioravanti and R. Tateo, Nucl. Phys. B **834**, 543 (2010) [arXiv:0912.4715 [hep-th]].

[33] N. Gromov and F. Levkovich-Maslyuk, JHEP **1006**, 088 (2010) [arXiv:0912.4911 [hep-th]].

[34] C. Krishnan, JHEP **0809**, 092 (2008) [arXiv:0807.4561 [hep-th]].

[35] B. I. Zwiebel, J. Phys. A **42**, 495402 (2009) [arXiv:0901.0411 [hep-th]].

[36] J. A. Minahan, O. Ohlsson Sax and C. Sieg, J. Phys. A **43**, 275402 (2010) [arXiv:0908.2463 [hep-th]].

[37] M. S. Costa, R. Monteiro, J. E. Santos and D. Zoakos, JHEP **1011**, 141 (2010) [arXiv:1008.1070 [hep-th]].

[38] R. A. Janik, P. Surowka and A. Wereszczynski, JHEP **1005**, 030 (2010) [arXiv:1002.4613 [hep-th]].

[39] K. Zarembo, JHEP **1009**, 030 (2010) [arXiv:1008.1059 [hep-th]].

[40] E. I. Buchbinder and A. A. Tseytlin, JHEP **1102**, 072 (2011) [arXiv:1012.3740 [hep-th]].

[41] E. I. Buchbinder and A. A. Tseytlin, JHEP **1008**, 057 (2010) [arXiv:1005.4516 [hep-th]].

[42] A. Grossardt and J. Plefka, arXiv:1007.2356 [hep-th].

[43] R. Roiban and A. A. Tseytlin, Phys. Rev. D **82**, 106011 (2010) [arXiv:1008.4921 [hep-th]].

[44] R. Hernandez, J. Phys. A **44**, 085403 (2011) [arXiv:1011.0408 [hep-th]].

[45] S. Ryang, JHEP **1101**, 092 (2011) [arXiv:1011.3573 [hep-th]].

[46] D. Arnaudov and R. C. Rashkov, Phys. Rev. D **83**, 066011 (2011) [arXiv:1011.4669 [hep-th]].

[47] G. Georgiou, JHEP **1102**, 046 (2011) [arXiv:1011.5181 [hep-th]].

[48] C. Park and B. -H. Lee, Phys. Rev. D **83**, 126004 (2011) [arXiv:1012.3293 [hep-th]].

[49] A. Bissi, C. Kristjansen, D. Young and K. Zoubos, JHEP **1106**, 085 (2011) [arXiv:1103.4079 [hep-th]].

[50] D. Arnaudov, R. C. Rashkov and T. Vetsov, Int. J. Mod. Phys. A **26**, 3403 (2011) [arXiv:1103.6145 [hep-th]].

[51] D. Arnaudov and R. C. Rashkov, Phys. Rev. D **83**, 066011 (2011) [arXiv:1011.4669 [hep-th]].

[52] X. Bai, B. -H. Lee and C. Park, Phys. Rev. D **84**, 026009 (2011) [arXiv:1104.1896 [hep-th]].

[53] B. -H. Lee and C. Park, Phys. Rev. D **84**, 086005 (2011) [arXiv:1105.3279 [hep-th]].

[54] C. Ahn and P. Bozhilov, Phys. Lett. B **702**, 286 (2011) [arXiv:1105.3084 [hep-th]].

[55] P. Bozhilov, JHEP **1108**, 121 (2011) [arXiv:1107.2645 [hep-th]].

[56] P. Bozhilov, Nucl. Phys. B **855**, 268 (2012) [arXiv:1108.3812 [hep-th]].

[57] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150].

[58] D. Z. Freedman, S. D. Mathur, A. Matusis and L. Rastelli, Nucl. Phys. B **546**, 96 (1999) [hep-th/9804058].

[59] V. Balasubramanian, P. Kraus, A. E. Lawrence and S. P. Trivedi, Phys. Rev. D **59**, 104021 (1999) [hep-th/9808017].